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On the Complexity of Reducing the Energy Drain in Multihop Ad Hoc Networks

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Abstract: Numerous studies on energy-efficient routing for Multihop Ad Hoc Networks (MANET) look at extending battery life by minimizing the cost at the transmitting node. In this paper, we study the complexity of energy efficient routing when the energy cost of receiving packets is also considered.

We first prove that, surprisingly, even when all nodes transmit at the same power, finding a simple unicast path that guarantees enough remaining energy locally at each node in the network then becomes an NP-complete problem.

Second, we define formally the problem of finding a virtual backbone that minimized the overall energy cost and prove that this leads to a new NP-complete problem, that we name Connected Exact Cover.

Finally, we provide a fully distributed algorithm to reduce the energy drain due to the number of redundant receptions in MANET protocols by offering a modification of the Multi-Point Relay selection scheme and give some provably optimal approximation bounds.

Key-words: Approximation Algorithms, Complexity, Energy-efficient routing, Mobile ad hoc, MultiPoint Relays, NP-completeness, Wireless Networks.

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Sur la complexité de la réduction des fuites d'énergie dans les réseaux ad hoc multi-sauts

Résumé : De nombreuses études sur le routage efficace, en terme d'énergie, dans les réseaux multi-sauts ad hoc (MANET) cherchent à rallonger la durée de vie des batteries en minimisant le coût énergétique de transmission aux noeuds émetteurs. Dans ce papier, nous étudions la complexité du routage efficace du point vue de la consommation d'énergie lorsque le coût énergétique aux noeuds récepteurs est également pris en compte.

Nous prouvons tout d'abord que, étonnament, même si tous les noeuds transmettent à la même puissance, trouver un simple chemin unicast qui guarantie une énergie résiduelle suffisante locallement à chaque noeud du réseau devient un problème NP-complet. Nous définissons également formellement le problème de la recherche d'une structure virtuelle qui minimise le coût énergétique global et prouvons que ce nouveau problème, appelé Connected Exact Cover, reste NP-complet.

Finalement, nous donnons un algorithme réparti pour réduire la perte d'énergie dûe aux réceptions redondantes des protocoles MANET en introduisant une variante de l'algorithme de sélection des relais multi-points et donnons des bornes d'approximations optimales.

Mots-clés : Algorithmes d'Approximations, Complexité, Conservation d'énergie, Mobile ad hoc, MultiPoint Relais, NP-complétude, Réseaux Sans Fils.

1 Introduction

Mobile Ad Hoc Networks (MANET) are unlike the well-studied cellular wireless systems that rely heavily on the robust structure of the physically connected stations. They are self organising entities that must distributedly choose how to interconnect in order to facilitate the communication within the network. This feature makes them attractive but increases the difficulty of the routing. A mobile node has to cooperate with other hosts to find routes and relay messages. In addition, mobile nodes have limited energy source (*i.e.,* battery) and communication range (each message may "hop" several times from node to node before reaching its destination).

In this work we are concerned with energy consumed by the MANET protocols as a performance metric. We are interested in characterising some of the energy-efficient features required by MANET protocols to route at minimum energy-cost. We focus our study on the fact that wireless traffic carried by neighbor links may interfere (contrarily to wireline networks for which each link is physically isolated from the other links even if they are attached to the same node). Hence, in this paper, we address the problem of energy-efficiency in MANET when the energy cost of packet receptions is also taken in consideration. We show that, even with simple assumptions such as a fixed or common transmission power, more problems become NP-hard, and some cannot be approximated in polynomial time.

It was already known that the energy-optimal broadcast is an NP-hard problem when nodes may choose among different transmission powers (*e.g.,* [4]). The first novel element brought in Section 3 is the fact that finding a simple path that guarantees enough remaining energy locally at each node is then an NP-complete problem (even when all nodes transmit at the same power).

In Section 4, we define formally the problem of finding a virtual backbone that minimized the overall energy cost when reception is considered and prove that this new problem, named Connected Exact Cover, is reminiscent of the Exact Set Cover problem and remains NP-complete. Finally, in Section 5, we provide a fully distributed algorithm to reduce the energy drain due to the number of redundant receptions in MANET protocols by offering a modification of the Multi-Point Relay selection scheme and give some provably optimal approximation bounds.

We introduce our motivations, model and notations in the following section.

2 Energy Efficiency in MANET

2.1 Motivation

The structureless and mobile nature of MANET nodes drastically limits their battery lifespan. Energy-efficient Multihop Ad Hoc Networks (MANET) look at extending battery life by minimizing the energy cost. The difficulty of such a problem lies on the fact that it involves several network layers. Three main approaches to alleviate this problem have been taken:

1. Modify the MAC layer (for IEEE 802.11).
2. Use a energy metric (*e.g.,* the energy consumed per packet or per node) and design schemes that minimize such a metric (at the physical or link layer).
3. Find an optimal transmit power locally at each node to control some global properties (*e.g.,* connectivity [7, 4], cone-based geometrical cover [22], constant stretch-factor of the shortest-path).

As some devices have the ability to adjust power, most of the theoretical work has been achieved for the last item and led to many NP-complete problems (*e.g.,* energy-optimal broadcast [7, 3]) and approximations heuristics (*e.g.,* broadcasting increment power [1]). There exist some implemented protocols that adaptively adjust power to optimize the network performance, by taking the smallest common power level which results in connectivity of the overall network [19]. Several studies have also suggested energy dependent participation protocols where nodes can evaluate and elect their involvement in the network based on the remaining level of their battery (*e.g.,* [20]).

To our knowledge, almost all studies have focused on transmitting energy as the sole energy cost. However, as pointed out in Feeney and Nilsson's seminal work [11] regarding the energy consumption of MANET current interfaces, there are other substantial costs. These are mainly due to the differences between MANET mode and base station mode. In MANET, there is no infrastructure and the nodes cannot go to an energy-saving ("sleep") mode as the intrinsic ad hoc nature of the network force them to remain in a "ready to receive" state (*i.e.,* the so-called "idle" state). Receiving any packet is costly, the conservation of energy could occur only after discerning that an incoming packet is not intended for a node. For instance, in [11], the Lucent IEEE 802.11 2Mbps WaveLAN PC Card power consumption characters were measured as follow: Idle power at 843mW, Receive power at 967mW, Transmit power at 1327mW.

As overall consumption is dependent of the mobile hardware, we restrict our study at considering "unit" of transmission and reception energy costs of the wireless network interface.

Currently there is a large body of work to reduce substantially this "idle state" cost (*e.g.,* at the MAC layer) as its rate of energy consumption is high and only slightly lower than the reception cost. Also, there has been work on how to schedule sleep periods among nodes that have limited battery level so that the network can still function and yet allow nodes to disconnect to save energy (*e.g.,* [5, 23]). It is expected that rapidly the difference of cost between reception and idle states will increase.

However the tenet that the reception cost is larger than a substantial fraction of the transmission cost is likely to remain. That is, defining ι_i, τ_i and ρ_i as the idle state, the transmission and reception interface energy costs for a node i, respectively, we can define $t_i(=\tau_i-\iota_i)$ and $r_i(=\rho_i-\iota_i)$ as the transmission and reception interface's energy *net* costs for a node i, respectively. Thus, we may assume that for some small integer constant k:

$$r_i > \frac{t_i}{k} \tag{1}$$

If k is smaller than the number of neighbors of a transmitting node, the passive receptions from non-targeted neighbors may cost more than the transmission from the source and the reception from the targeted neighbors. It can lead to a detrimental drain of nodes with low battery even if these did not need to be involved in the transmission.

In this paper, we study the impact of link interferences on energy efficient routing in MANET by considering the energy cost of transmitting **and receiving** packets (even when all nodes transmit at the same power). For the rest of the paper, we will assume that t_i and r_i are fixed at each node i.

2.2 Model and notations

For clarity, and without loss of generality, we introduce a simple graph model (none of the simplistic assumptions change the impact of our complexity results). We introduce several notations. For other basic graph-theoretical definitions we refer the reader to Diestel [9].

We consider an undirected graph $G(V, E)$ modeling a wireless network. Link (i, j) means that nodes i and j can communicate. We assume that all links are bidirectional. Node i has *remaining energy capacity* e_i and can send a message to its neighbors at *transmitting interface's energy cost* t_i if $t_i \leq e_i$.

Let $N(j)$ be the neighbors of node j. Let $N^2(j)$ denote the two-hop neighbors of j (the nodes neighbors of the neighbors of j which are not already neighbors of j). Denote by $N_j(h)$ the set of nodes that are at most h hops away from node j (including node j), *e.g.,* $N_j(1) = \{\{j\} \cup N(j)\}$.

Let Δ denote the maximum degree of a node in the graph (*i.e.,* $\Delta = \max_{u \in V} |N(u)|$). Let $d_u^+(v) = |\{w \in N(v) | v \in N(u) \text{ and } w \in N^2(u)\}|$, that is the number of neighbors of a neighbor v of u that are two-hop away from u. Let Δ_u^+ denote $\max_{v \in N(u)} d_u^+(v)$.

A transmission interferes with nodes that are within a number of hops, H_I, from the transmitter, depending on the signal to noise ratio required for a correct reception. When node i transmits a packet to its neighbor j, all nodes in $N_i(H_I)$ can receive the packet and may need to process it. Therefore consuming t_i units of interface energy (at node i) for transmission from i to j induces the consumption of r_k units of interface energy at each receiving node k within H_I hops from i.

In this context, it is easy to compute the worst-case *per-packet cost* for passing a packet along a path Π from a source s to a destination t:

$$\sum_{i \in \Pi - \{t\}} (t_i + \sum_{j \in N_i(H_I)} r_j) \tag{2}$$

Let $K_\Pi(i) = \{j \in \Pi - \{t\} | i \in N_j(H_I)\}$, that is, the nodes on the path Π that are within H_I hops from i. Let $k_\Pi(i) = |K_\Pi(i)|$. The *per-node cost* at a node i (for passing a packet along a path Π from a source s to a destination t, possibly not including i) is defined as:

$$\begin{array}{lll} r_i\ k_\Pi(i) & + t_i & \text{if } i \in \Pi, \\ r_i\ k_\Pi(i) & & \text{otherwise.} \end{array} \tag{3}$$

The second case formalizes the fact that the battery of a node in the vicinity of some transmissions may be drained rapidly without transmitting once.

This emphasizes also the fact that the *per-packet cost* and *per-node cost* lead to two distinct problems. When all the nodes transmit at the same energy cost, it is easy to find a unicast path that minimize the per-packet cost (*e.g.,* by using a Dijkstra's shortest path algorithm running in polynomial time). However minimizing the per-node cost is not trivial and is in fact NP-complete if some minimum remaining energy is required as we will show in the next section.

3 Per-node energy efficiency and NP-completeness

Several studies have suggested energy dependent participation protocols where nodes can evaluate and elect their involvement in the network based on the remaining level of their battery (e.g., [20]). Within our model, this raises the natural question of avoiding the drain of the energy of nodes that are not really necessary for the multihop communication.

This yields to the following remaining energy problem.

3.1 Local minimum remaining energy

We may want to ensure that a flow is not detrimental to the remaining energy of some neighboring low-level nodes. We say that the flow path is tolerable if this operation leaves the remaining energy e_j of all nodes j in the network to a minimum tolerable capacity level c_j. (For the critical functioning of each individual node, one may need that the remaining energy is always above a non-zero fraction of the nominal capacity.)

Assume that node i has minimum tolerable capacity c_i. We must have

$$c_i \leq e_i, \ i \in V \tag{4}$$

We can now define formally the energy-efficient "Path with Remaining Energy problem" as follows.

Definition 1 *The Path with Remaining Energy problem (RE) is defined as:*
Instance*: A Graph $G = (V, E)$, two vertices s and t from V, the remaining energy e_i and a minimum tolerable capacity c_i, both in $\mathbb{R}$, for each vertex i from V.*
Question*: Is there a simple path from s to t in G that satisfies the constraint $c_i \leq e_i, \ i \in V$?*

In the following section, we prove that, because of the basic constraint (4), this problem is NP-complete.

3.2 NP-completeness proof

We first recall the definition of the *Path with Forbidden Pairs problem (PFP).*

Definition 2 *The Path with Forbidden Pairs problem (PFP)(see GT54 in [12]) is defined as:*
Instance: *A Graph $G = (V, E)$, two vertices s and t from V, and a collection $C = \{(a_1, b_1), \dots, (a_m, b_m)\}$ of pairs of vertices from V.*
Question: *Is there a simple path from s to t in G that contains at most one vertex from each pair in C?*

The PFP problem is known to be NP-complete (see GT54 in [12]). Variants of this problem exist where a measure is the length of the path (*i.e.,* the number of edges in the path): the shortest feasible PFP and the longest feasible PFP problems. Not surprisingly both variants are NP-complete and also NPO PB-complete [14], *i.e.,* in polynomial time, they cannot be approximated within n^ε for some $\varepsilon > 0$, where n is the size of the input, provided that $P \neq NP$.

Theorem 3 *The Path with Remaining Energy problem (RE) is NP-complete.*

Proof. For clarity, we first prove the theorem in the case $H_I = 1$ (*i.e.,* only neighbors of the node are receiving the packets transmitted by this node).

It is easy to see that $RE \in NP$, because a nondeterministic algorithm needs only to guess a path and check in polynomial time that the constraint $c_i \leq e_i,\ i \in V$, is valid. (Of course, checking the constraint for the vertices of the path and for their neighbors is sufficient.)

We now give a polynomial reduction from this problem to the Path with Forbidden Pairs problem (PFP).

Without loss of generality, we assume that for any node, a reception costs one unit of energy (*i.e.,* $r_i = 1$ for any node i). We first transform an instance $(G = (V, E), s, t, C)$ of the PFP problem in an instance $(G' = (V', E'), s, t, p, c)$ of the Remaining Energy problem by formally defining:
- $V' = V \cup \{v_{xy} | (x, y) \in C\}$,
- $E' = E \cup \{(x, v_{xy}), (y, v_{xy}) | (x, y) \in C\}$,
- s and t are unchanged,
- c_i, the minimum tolerable capacity at any node i is set to an arbitrary non-negative value,
- e_i, the remaining energy set to:

$$\begin{array}{ll} e_i = c_i, & \text{if } i \in \{v_{xy} | (x, y) \in C \text{ and } x = t \text{ or } y = t\}, \\ e_i = c_i + 1, & \text{if } i \in \{v_{xy} | (x, y) \in C \text{ and } x \neq t \text{ and } y \neq t\}, \\ e_i = c_i + |V|, & \text{otherwise.} \end{array}$$

Informally, G' contains all the vertices of G as well as m vertices, each representing a forbidden pair. Let us define F as the set of m vertices in G' representing each forbidden pairs of C. Each vertex of F is only connected to its two respective "forbidden" vertices and is assigned an actual energy equal to 1 more than the capacity (*i.e.,* only one "forbidden" node can transmit) or to the capacity (if the destination is part of the forbidden pair). All other vertices are assigned the remaining energy equal to the capacity plus $|V|$, that is the remaining energy is larger than any possible energy cost induced by a given established path.

We now prove that a solution of this instance of the RE problem is a solution if and only if it is a solution for the original instance of the PFP problem.

It is easy to see that a solution path Π from s to t for the RE problem in $(G' = (V', E'), s, t, p, c)$ does not include any of the vertices of F, as each vertex v of the path (except the destination t) requires to decrement e_v by at least 2. Hence this path Π is also a path Π in G.

Furthermore, none of the forbidden pairs are included in the solution path. Otherwise there would exist such a pair of vertices a and b (with $(a, b) \in C$) that both belong to the solution path. This would imply that the vertex x_{ab} receives more than 2 transmissions along the path within 1 hop and that the basic energy constraint for the vertex x_{ab} is not valid as it will decrement $e_{x_{ab}}$ by 2 and make it smaller than $c_{x_{ab}} = 1$, thus leading to a contradiction. In the case that one of the forbidden pairs' node is the destination, the other cannot transmit without decreasing the remaining energy $e_{x_{ab}}$ by 1 which is then smaller than $c_{x_{ab}} = 0$, also leading to a contradiction. Hence a solution for the RE problem in $(G' = (V', E'), s, t, p, c)$ is a solution for the instance $(G = (V, E), s, t, C)$ of the PFP problem.

Conversely, given a solution path Π' for the instance $(G = (V, E), s, t, C)$ of the PFP problem, we can verify that the path Π' is a feasible solution path for the RE problem in $(G' = (V', E'), s, t, p, c)$. Obviously, Π' is a simple path of G' as G is a subgraph of G'. Hence, we just need to verify that the remaining energy induced by the path respect the energy constraints. It is clear from the reduction that only the nodes of F may jeopardize the feasibility of the solution, as they may not have sufficient energy. Again, none of them belong to the path. A vertex of F can be a neighbor of a vertex of Π', incurring an energy cost of at least 1 (unless it is a neighbor of the destination). However, a vertex of F cannot be a neighbor of two nodes in the paths as it can only be neighbor of its forbidden pair which would contradict the feasibility of Π' for the instance $(G = (V, E), s, t, C)$ of the PFP problem.

As the proof follows a polynomial reduction from this problem to the Path with Forbidden Pairs problem (PFP), the Path with Remaining Energy problem (RE) is NP-complete with $H_I \geq 1$. (The case $H_I = 0$ corresponds to the case where a node cannot hear any neighbor.) □

This theorem proves that looking for "any" path that satifies the basic constraint 4 is not tractable. For other considerations, we may also request that each path reserved is a shortest path. Using the result of Kann [14] on the shortest feasible PFP, it is easy to deduce the following corollary.

Corollary 4 *Shortest Path with Remaining Energy problem ($SPRE$) remains NP-complete and is also hard to approximate (i.e., NPO PB-complete).*

We could introduce some variants of these optimization problems while respecting the basic constraint (*e.g.,* Least Remaining Energy, Total Consumed Remaining Energy), but all are NP-complete and one need to introduce some heuristics. It is obvious that, even with a more detailed reception model or with localization, the NP-completeness of each problem still holds.

4 Virtual Backbone and Reception-Awareness

For reasons such as scaling, it may be interesting to "virtually" structure the ad hoc network hierarchically. In the past decade, numerous topology structures were suggested: clusters, trees, spanners,... Minimum Spanning Trees and Spanners in general (*e.g.,* [1]) are simple enough to limit the control of specific nodes while maintaining some important global properties in the induced graph (*e.g.,* connectedness, constant stretch-factor compared to the shortest-path in the existing network). However, when more than one parameter need to be optimized, advanced topologies are required.

For instance, clustering mobile nodes locally is an effective way to hierarchically organise the virtual structure. Minimizing the number of clusters to optimize the traffic control is a well-known combinatorial problem called the Minimum Dominating Set problem (DS) and is NP-complete [12]. This scheme can be refined to Connected Dominating Set (CDS) or Weakly-Connected Dominating Set ($WCDS$) [6] to allow straightforward routes between different clusters, and hence form a connected induced component (*i.e.,* a *virtual backbone*). Again, the size of the backbone must be minimum and these problems remain NP-complete.

Virtual backbone always leads to a well-known NP-complete combinatorial problem and approximations algorithms that can be fully distributed are then required. In this section, we will show that considering the reception energy cost when building a virtual backbone also leads to an NP-complete problem related to a well-known combinatorial problem.

4.1 Backbone and Minimum set cover

It is easy to see that all virtual backbone problems are reminiscent of the NP-complete Minimum Set Cover problem [12], defined as follows.

Definition 5 *Minimum Set Cover (SC) is defined as:*
Instance*: A Collection C of subsets of a finite set S and an integer B.*
Question*: Is there a set cover for S, i.e., a subset $C' \subseteq C$ such that every element in S belongs to at least one member of C', such that $|C'| \leq B$?*

For example, it is easy to see that the Dominating Set problem is a particular case of SC: for each node u of the graph, assign a subset in C for each node and its neighbors (*i.e.,* $C = \{S_u | S_u = \{u \cup N(u)\}, \forall u \in V\}$).

Two approximations results of interest are known for the SC problem (hence for the dominating set variants):

1. The SC problem is approximable within $1 + \ln |S|$ [16]. Unfortunately, it is also known that it is not approximable within $(1 - \varepsilon) \ln |S|$ for any $\varepsilon > 0$, unless NP $\subset \textsc{Dtime}(n^{\log \log n})$ [10].

2. When the size of each subset of C is bounded by a constant Δ independent of the size of the input, it is approximable within $H(\Delta) = \sum_{j=1}^{\Delta}(1/j)$ (the Harmonic function) [16]. This ratio is slightly less than $1 + \ln \Delta$ (as $H(\Delta) < 1 + \ln \Delta < H(\Delta) + \frac{1}{2}$).

Such bounds are achievable by running an algorithm that follows a simple degree-greedy strategy algorithm. In particular, the $H(\Delta)$ approximation ratio is tight as there exist graphs (with sufficiently large $|S|$) for which such an algorithm will attain such ratio [16].

It should be noted that it makes sense to assume that, whatever the wireless technology used, the number of neighbors each node can "communicate" with is upper bounded by a constant independent of the size of the network. Although theoretically the maximum degree of a wireless node could be dependent of the size of the network, a recent study on the capacity of wireless networks [13] shows that it will remain small in practice. In fact, with the current technology, Δ can reach possibly thousands in theory but only few dozens in practice. Hence, the approximation factors will remain small (*i.e.,* $1 + \ln \Delta$ will reach 8 or 3, respectively). Recently, a heuristic approach for producing a low-energy topology of bounded degree [21] and a protocol maintaining a constant number of neighbors has been proposed [2].

4.2 Backbone with reception-awareness

When the number of reception is considered (to minimize their energy cost), one would like to obtain a virtual backbone with as few as possible cover sets but these should "cover" disjoint sets, if possible. Informally, the goal is to build a Connected Dominating Set (CDS) of small size where non-dominating nodes have as few as possible dominating neighbors.

It could be easy to introduce a weighted version of a CDS heuristic to take in account the number of triggered receptions by a transmissions at a node. Each node i can locally compute the maximal possible reception energy cost by summing the cost at all its neighbors and set its weight w_i to $t_i + \sum_{j \in N_i(H_I)} r_j$. The quality of a weighted solutions where the energy-cost is taken as the metric (as per node-basis) obtained an SC algorithm can be done easily and will obtain the same approximation performance [16] (although independent of the weight function chosen). However, this does not directly reduce the number of unnecessary receptions at any individual node. A good solution for the overall network may be fatal for several nodes that will be drained of their energy without communicating.

Hence another possibility is to modify the combinatorial problem objective altogether by minimizing the number of overlaps between selected cover sets. If the connectivity is not required, this minimization problem is known as the Minimum Exact Cover and is NP-complete. It is defined formally as follows.

Definition 6 *Minimum Exact Cover (EC) is defined as:*
Instance*: A Collection C of subsets of a finite set S and an integer B.*
Question*: Is there an exact cover for S, i.e., a subset* $C' \subseteq C$ *such that every element in S belongs to at least one member of C', such that* $\sum_{c \in C'} |c| \leq B$?

This is approximable within $1 + \ln|S|$ [16] but as hard to approximate as SC [18]. Also the same approximation ratio is achievable by a weighted version of this problem (where the objective is to minimize the sum of the weights in the set cover and where the weights are counted as many time as that they are covered). Although, the only difference between SC and EC is the definition of the objective function, our proposal is not a slight formal modification: the two problems may diverge. The best approximation heuristic known for SC may obtain an optimal solution for a given instance of the SC problem while obtaining an extremely poor solution for the same given instance for the EC problem: the approximation factor could be as bad as $O(|S|)$.

A simple heuristic with good approximation ratio is known [18]. The idea of the algorithm is to limit the overlapping by greedily selecting nodes that minimize their respective ratio of the number of already covered nodes over the number of uncovered nodes. In the next section, we adapt this heuristic to provide a fully distributed algorithm for broadcast.

It is now easy to see that a connected backbone with reception-awareness (*i.e.,* a CDS of small size where non-dominating nodes have as few as possible dominating neighbors) can be defined formally as follows. (As it is unknown to us, we name it the Connected Exact Cover problem.)

Definition 7 *Connected Exact Cover (CEC) is defined as:*
Instance*: A Graph $G = (V, E)$ and an integer B.*
Question*: Define a finite set $S = V$ and, for each node u of the graph, assign a subset S_u in C including the node u and its neighbors (i.e., $C = \{S_u | S_u = \{u \cup N(u)\}, \forall u \in V\}$). Define u as the central node of S_u. Is there an exact cover $C' \subseteq C$ for S such that the central nodes of C' form a connected components and $\sum_{c \in C'} |c| \leq B$?*

As this problem is a special instance of the Exact Set Cover problem, it is easy to prove that it remains NP-complete.

Theorem 8 *The Connected Exact Cover problem (CEC) is NP-complete.*

5 Distributed Algorithms for MANET

In this section, we provide a natural algorithm to minimize the number of receptions induced by the routing protocol by adapting adequately a commonly used broadcast scheme. This broadcast mechanism uses specific relay nodes called MultiPoint Relay nodes (MPR). The MPR scheme was first introduced in [15] and used for MANET protocols (*e.g.,* [8]).

The IETF standardization forum is addressing the problem of mobile ad hoc routing in the working group MANET. Several protocols have been proposed in experimental standards. Energy conservation schemes are not usually part of the IETF protocols and only subtle modifications to the existing core of each protocol need to be achieved in order to avoid jeopardizing the overall protocol.

Currently there are two main distinct classes of routing protocols: proactive and reactive. When the traffic is low, reactive protocols are intrinsically saving energy by limiting their control traffic to a minimalist on-demand scheme. Proactive protocols maintain up-to-date routing information by propagating updates throughout the network and mainly limit the energy consumption of the nodes by minimizing the cost incurred by their broadcast schemes. Some proactive protocols (such as OLSR [8]) are well-suited for the energy control because they contains an embedded MPR broadcast mechanism that minimizes the number of retransmissions.

In this section, we introduce a different MPR selection strategy to reduce the number of receptions (and this modification is of interest on its own). To illustrate the importance of the Multi-Point Relay concept, we briefly introduce its use in the OLSR protocol for MANET. We limit our presentation to the basic algorithmic point-of-view necessary for the energy-efficiency study. Readers interested in further details should read the experimental IETF-RFC3626 protocol document [8]. With reception minimization in mind, we then introduce a modified MPR selection algorithm that could be implemented without jeopardizing any protocol that already use such scheme.

5.1 MPR Selection and Complexity

The goal of Multi-Point Relays is to reduce the flooding of broadcast packets in the network by minimizing the duplicate retransmissions locally. Each node selects a subset of neighbors called Multi-Point Relays (MPRs) to retransmit broadcast packets. This allows neighbors which are not in the MPR set to read the message without retransmitting it, this prevents the flooding of the network (*i.e.,* the so-called *broadcast storm*). Of course, each node must select an MPR set among its neighbors that guarantees that all two-hop away nodes will get the packets, *i.e.,* all two-hop away nodes must be a neighbor of a node in the MPR set (see Figure 1, where MPR nodes are in red/grid).

In the OLSR protocol, each node periodically broadcast the information about its immediate neighbors which have selected it as an MPR. Upon receipt of this information, each node calculates and updates its routes to each destination (*i.e.,* the sequence of hops through the successive MPRs from source to destination). Notice that the neighbor discovery overhead is unchanged and its locality makes easy to implement distributedly in an efficient way. The MPR flooding still follows a simple rule: a node retransmits a broadcast packet if and only if it was received the *first* time from a node for which it is an MPR. The main gain obtained by introducing MPRs is that: the smaller the MPR set, the smaller the number of packet retransmissions.

For example, in Figure 1 only three out of the ten neighbors of the source may retransmit the packets, and this MPR set is of minimum size.

Several important properties can be proved about this scheme [15]. In particular, the use of MPRs (instead of all the neighbors) does not destroy the connectivity properties of

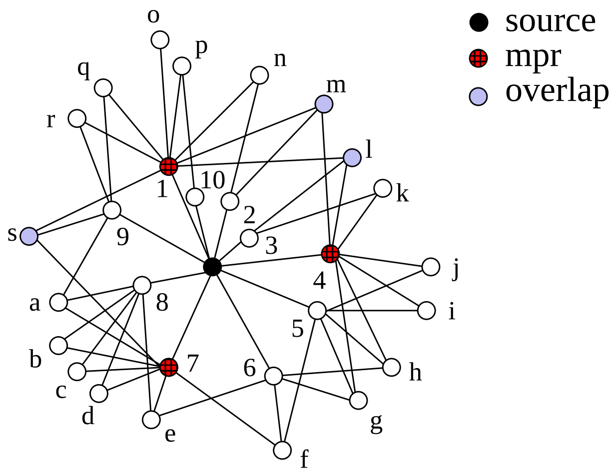

Figure 1: MPR Selection.

the network and MPRs provide shortest-path routes for unicast with respect to the original graph. Several polynomial-time heuristics were proposed to select an MPR set of minimal cardinality at each node as the network topology can be arbitrary and it was proven that the selection of a minimum size MPR set is NP-complete (*e.g.,* [17]) by reduction to the Minimum Domination Set problem.

Using our notations, an MPR set of a node u is a subset $MPR(u)$ of $N(u)$ such that:

$$\forall w \in N^2(u), \exists v \in MPR(u) \text{ such that } w \in N(v).$$

Let $MPR^*(u)$ denote an MPR set of minimum cardinality for a node u.

Definition 9 *Minimum Multi-Point Relay (MPR) is defined as:*
Instance*: A network G (defined as a graph $G(V, E)$), a node u of $V(G)$ and an integer B.*
Question*: Is there a Multi-Point Relay $MPR(u)$ set of u of size less than B?*

Again, it is easy to see that this MPR problem is essentially the same as the Minimum Set Cover problem. Let $S = N^2(u)$. We assign a subset in C with each neighbor of u: $C = \{S_v | \forall v \in N(u), S_v = \{w \mid w \in N^2(u) \text{ and } w \in N(v)\}\})$.

The approximation bounds presented in the previous section are easily achievable. Except for the initial phase, the MPR algorithm currently used in the OLSR protocol implementation is similar to the "degree-greedy" algorithm presented in [16]). For any node u: first select as MPR the neighbors of u that are the only neighbors of some two-hop nodes from u; second, while there are still some uncovered two-hop nodes from u, select as MPR a neighbor of u that is neighbor to the largest number of remaining uncovered two-hop nodes.

The first phase is added to adequately use the fact that, whatever the strategy chosen, one-hop nodes that are the sole "cover" of two-hop nodes must be included in the MPR set.

For example, in Figure 1, node o has only one neighbor among the neighbors of the source; hence this respective neighbor (node 1) must be included in the MPR set. It is clear that this slight modification yields better-in-practice solutions without weakening the approximation bound [17].

In the MPR wireless context, it implies that it is easy to design heuristics that selects an MPR set with the respective performance approximation ratio:

1. $1 + \ln |S| = 1 + \ln |N^2(u)|$ in the general case and,
2. $1 + \ln \Delta_u^+$, when Δ_u^+ (the maximum number of two-hop nodes that each one-hop neighbors of u may cover) is bounded by a constant independent of the size of the network.

5.2 MPR Selection with Minimum Overlapping.

The quality of a weighted solution where the energy-cost is taken as the metric (as per node-basis) obtained by the previous algorithm can be done easily and will obtain the same approximation performance.

Again, a better approach is to minimize the MPR selection objective altogether to reduce the overlapping of the cover of two-hop nodes by MPR nodes thus reducing the number of redundant receptions.

Indeed, as defined the MPR problem aims at reducing the cardinality of MPR set without considering its topology. The topology of the MPR set, *i.e.,* the way the MPR nodes are positioned from one another will have a real impact on the behaviour of the routing while considering other problems, such as collision avoidance or available remaining energy. For example, the principal role of the MPRs during the broadcast phase is to forward packets effectively with reduced duplication in order to limit the traffic and the risk of collisions. In Figure 1, the size of MPR set is minimum, however pair of nodes 1, 4 and 7 cover nodes k, l and s. Such overlap will be detrimental to some applications as known problems such as the so-called *Hidden Terminal* may occur and packets may be lost.

Ideally, one would like to obtain small MPR set for which MPR nodes "cover" disjoint sets of two-hop nodes. Of course, this may be impossible to achieve if two neighbors of the source overlap in their covers and are each the sole cover of a respective two-hop node. For example, in Figure 1, if node 4 was also sole neighbor of a two-hop neighbor, we could not avoid including 4 as an MPR node and both nodes k and l would have to be "overlapped" by the cover of nodes 1 and 4. This unfortunate case may involve an arbitrarily large number of nodes (up to the maximum degree of the nodes minus one). Hence, it is pointless to expect to reduce the maximum number of overlapping per node.

However, it is possible to limit the impact of the overall overlapping according to a given source, *i.e.,* the least overall overlapping of the MPR set. Using the same topology as Figure 1, one could choose a MPR set that reduces the number of nodes that "overlap" from 3 to 1, by incrementing the size of the MPR set (see Figure 2).

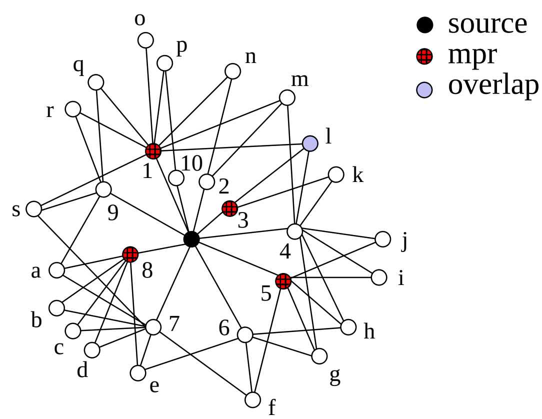

Figure 2: MPR Selection with minimum overlapping.

Again, this minimization problem is the Minimum Exact Cover, hence the same bound and the same proof apply. As for the previous MPR algorithm, the major difference is the introduction of an initial phase that includes immediately the one-hop nodes that are the sole neighbor of a two-hop neighbor into the MPR set. For completeness we describe the algorithm for any node u:

1. Select as MPR the neighbors of u that are the only neighbors of some two-hop nodes from u;
2. While there are still some uncovered two-hop nodes from u:
   select as MPR a neighbor of u that has the least ratio of the number of remaining uncovered two-hop nodes over the number of covered two-hop nodes (if ties exist, select the node with maximum number of uncovered nodes).

In the case of Figure 1, the algorithm will successively selects as MPR nodes 1, 8, 5 and 3. Using [16], we can immediately deduced the two following corollaries and one can build a graph for which such a bound is attainable.

Corollary 10 *The MPR Selection Algorithm for limited overlapping guarantees an approximation ratio to the optimal size of* $1 + \ln |N^2(u)|$ *for a source node* u.

Corollary 11 *When* Δ_u^+ *is bounded by a constant independent of the size of the network, the MPR Selection Algorithm for limited cover overlapping guarantees an approximation ratio of* $1 + \ln \Delta_u^+$ *for a source node* u.

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